Due Date: 23.10.2019 - Monday - Morning 10:00 AM

HOMEWORK 2

Que: (30 p) Assume that $x, y \in R$ and $z = x + iy \in C$ and answer the followings:

1. Find the values of sin i, cos i, tan (1 + i).

2. The hyperbolic cosine and sine are defined by $\cosh z = (e^z + e^{-z})/2$, $\sinh z = (e^z - e^{-z})/2$. Express them through $\cos iz$, $\sin iz$. Derive the addition formulas, and formulas for $\cosh 2z$, $\sinh 2z$.

3. Use the addition formulas to separate $\cos (x + iy)$, $\sin (x + iy)$ in real and imaginary parts.

4. Show that

$$|\cos z|^2 = \sinh^2 y + \cos^2 x = \cosh^2 y - \sin^2 x = \frac{1}{2} (\cosh 2y + \cos 2x)$$

and

$$|\sin z|^2 = \sinh^2 y + \sin^2 x = \cosh^2 y - \cos^2 x = \frac{1}{2} (\cosh 2y - \cos 2x).$$

Que: (30 p) Assume that $x, y \in R$ and $z = x + iy \in C$ and answer the followings:

- **3.** Find the value of e^z for $z = -\frac{\pi i}{2}, \frac{3}{4}\pi i, \frac{2}{3}\pi i$.
- 4. For what values of z is e^{z} equal to 2, -1, i, -i/2, -1 i, 1 + 2i?
- 5. Find the real and imaginary parts of exp (e^z) .

Que: (30 p) Assume that $x, y \in R$ and $z = x + iy \in C$ and answer the followings:

- 6. Determine all values of 2^i , i^i , $(-1)^{2i}$.
- 7. Determine the real and imaginary parts of z^{z} .
- 8. Express arc tan w in terms of the logarithm.

Que: (10 p) Using Cauchy Riemann equations prove that the followings do not have derivatives: (Assume that $x, y \in R$ and z = x + iy, $\overline{z} = x - iy \in C$)

(a)
$$f(z) = \overline{z}$$
; (b) $f(z) = z - \overline{z}$; (c) $f(z) = 2x + ixy^2$; (d) $f(z) = e^x e^{-iy}$.