## HOMEWORK 2

Que: (30 p) Assume that $x, y \in R$ and $z=x+i y \in C$ and answer the followings:

1. Find the values of $\sin i, \cos i, \tan (1+i)$.
2. The hyperbolic cosine and sine are defined by $\cosh z=\left(e^{z}+e^{-z}\right) / 2$, $\sinh z=\left(e^{z}-e^{-z}\right) / 2$. Express them through $\cos i z, \sin i z$. Derive the addition formulas, and formulas for $\cosh 2 z, \sinh 2 z$.
3. Use the addition formulas to separate $\cos (x+i y)$, $\sin (x+i y)$ in real and imaginary parts.
4. Show that

$$
|\cos z|^{2}=\sinh ^{2} y+\cos ^{2} x=\cosh ^{2} y-\sin ^{2} x=\frac{1}{2}(\cosh 2 y+\cos 2 x)
$$

and

$$
|\sin z|^{2}=\sinh ^{2} y+\sin ^{2} x=\cosh ^{2} y-\cos ^{2} x=\frac{1}{2}(\cosh 2 y-\cos 2 x) .
$$

Que: (30 p) Assume that $x, y \in R$ and $z=x+i y \in C$ and answer the followings:
3. Find the value of $e^{2}$ for $z=-\frac{\pi i}{2}, \frac{3}{4} \pi i, \frac{2}{3} \pi i$.
4. For what values of $z$ is $e^{z}$ equal to $2,-1, i,-i / 2,-1-i, 1+2 i$ ?
5. Find the real and imaginary parts of $\exp \left(e^{z}\right)$.

Que: (30 p) Assume that $x, y \in R$ and $z=x+i y \in C$ and answer the followings:
6. Determine all values of $2^{i}, i^{i},(-1)^{2 i}$.
7. Determine the real and imaginary parts of $z^{z}$.
8. Express arc $\tan w$ in terms of the logarithm.

Que: (10 p) Using Cauchy Riemann equations prove that the followings do not have derivatives:
(Assume that $x, y \in R$ and $z=x+i y, \bar{z}=x-i y \in C$ )
(a) $f(z)=\bar{z}$;
(b) $f(z)=z-\bar{z}$;
(c) $f(z)=2 x+i x y^{2}$;
(d) $f(z)=e^{x} e^{-t y}$.

